

A HARVESTED PREDATOR-PREY MODEL WITH PREY REFUGE UNDER UNCERTAIN ENVIRONMENTS

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Abstract: This study develops a predator-prey management model that integrates species-specific harvesting, a refuge mechanism jointly associated with prey and predator densities, and environmental parameter uncertainty. Triangular fuzzy numbers represent uncertainty in the prey intrinsic growth rate, predator mortality, and conversion efficiency. The ω -cut and utility-function procedures yield a deterministic model for equilibrium and local stability analysis. The analysis identifies the feasibility conditions for trivial, axial, and positive interior equilibria. Numerical experiments evaluate the effects of fuzzy weights, the uncertainty level, refuge intensity, and harvesting effort. The results indicate that appropriate refuge promotes stable coexistence, whereas excessive harvesting can destabilize coexistence or induce population loss. The proposed analytical scheme provides a basis for evaluating harvesting strategies when ecological parameters are imprecisely known.

Keywords: Fuzzy biological parameters; Predator-prey system; Stability; Harvesting; Prey refuge

1 INTRODUCTION

Predator-prey interaction is a central topic in mathematical ecology. The Holling type-II functional response is frequently used to represent saturation in predation, while prey refuge is recognized as an important determinant of coexistence and stability [1-2]. A refuge protects part of the prey population from predation and can substantially alter the qualitative behavior of a predator-prey system. Earlier analyses commonly used constant or prey-dependent refuge functions [1]. Subsequent work considered a refuge dependent on both prey and predator densities, which reflects the role of encounter-related ecological conditions in refuge formation [3-4].

Harvesting is also a consequential external intervention in renewable-resource systems. Bioeconomic management requires a balance between ecological persistence and harvesting returns [5]. Existing work has extended optimal-harvesting analysis to predator-prey systems with prey refuge and species-specific harvesting efforts [3, 6]. However, deterministic formulations generally presume that all biological parameters are known precisely. This assumption is restrictive when environmental variation, measurement error, and limited observations affect the intrinsic growth rate, predator mortality, and conversion efficiency [6-7].

Triangular fuzzy numbers provide a tractable representation of such imprecision. Prior studies have examined fuzzy predator-prey systems with refuge, fishery protection, optimal harvesting, fear effects, and related bioeconomic mechanisms [7-12]. Building on this literature, the present study constructs a harvested predator-prey model in which the refuge term depends on both species and selected biological parameters are fuzzy. The analytical framework combines ω -cut sets with a utility-function defuzzification procedure, derives equilibrium and local-stability conditions, and uses source-data simulations to examine parameter sensitivity. The resulting scheme is organized around model construction, equilibrium feasibility, stability criteria, and scenario-based assessment of uncertainty, refuge, and harvesting effects.

2 FUZZY PARAMETER REPRESENTATION

Let a triangular fuzzy number be denoted by $\tilde{U} = (u_1, u_2, u_3)$. Its membership function is represented by Equation (1), and its ω -cut set is a closed interval. For a given collection of items U_i with weights w_i , the utility function in Equation (2) aggregates the items according to their relative importance, where the weights are nonnegative and sum to one.

$$\mu_{\tilde{U}}(u) = \begin{cases} \frac{u-u_1}{u_2-u_1} & \text{if } u_1 \leq u \leq u_2 \\ \frac{u_3-u}{u_3-u_2} & \text{if } u_2 \leq u \leq u_3 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$U = \sum_{i=1}^n w_i U_i, w_i \geq 0, \sum_{i=1}^n w_i = 1 \quad (2)$$

For the fuzzy number $\tilde{A}$, the ω -cut set is $\tilde{A}_\omega = \{x : \mu_{\tilde{A}}(x) \geq \omega\}$. In the triangular case, the lower and upper endpoints are $U_l(\omega) = u_1 + \omega(u_2 - u_1)$ and $U_r(\omega) = u_3 + \omega(u_3 - u_2)$, respectively. These definitions are used to represent uncertain ecological parameters without replacing their biological interpretation.

3 MODEL CONSTRUCTION AND ANALYTICAL SCHEME

3.1 Harvested Predator-Prey System

Let x and y denote prey and predator population sizes. The prey population follows logistic growth with intrinsic growth rate r and carrying capacity k . The predation term has a Holling type-II form. A proportion mxy is protected by refuge, leaving $(1 - my)x$ prey exposed to predation. Species-specific harvesting is represented by q_1E_1x and q_2E_2y . The model is stated in Equation (3).

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1 - \frac{x}{k}\right) - \frac{p(1-my)xy}{1+ax(1-my)} - q_1E_1x \\ \frac{dy}{dt} &= -dy + \frac{ep(1-my)xy}{1+ax(1-my)} - q_2E_2y\end{aligned}\quad (3)$$

Here r, k, p, m, d, q_1 , and q_2 are positive biological or harvesting parameters. The conversion efficiency satisfies $0 < e < 1$. The harvesting efforts E_1 and E_2 are nonnegative. The refuge constraint $y \leq 1/m$ ensures that $0 \leq 1 - my \leq 1$ within the biologically feasible region.

3.2 Defuzzification Procedure

Environmental imprecision is assigned to the prey growth rate, predator mortality rate, and conversion efficiency as $\tilde{r} = (r_1, r_2, r_3)$, $\tilde{d} = (d_1, d_2, d_3)$, and $\tilde{e} = (e_1, e_2, e_3)$. Applying the ω -cut representation and the utility-function method yields the effective parameters in Equation (4) and (5).

$$\hat{r} = w_1r_1(\omega) + (1 - w_1)r_r(\omega), \hat{d} = w_2d_r(\omega) + (1 - w_2)d_l(\omega), \hat{e} = w_2e_l(\omega) + (1 - w_2)e_r(\omega) \quad (4)$$

$$\begin{aligned}\frac{dx}{dt} &= \hat{r}x\left(1 - \frac{x}{k}\right) - \frac{p(1-my)xy}{1+ax(1-my)} - q_1E_1x = F_1(x, y) \\ \frac{dy}{dt} &= -\hat{d}y + \frac{\hat{e}p(1-my)xy}{1+ax(1-my)} - q_2E_2y = F_2(x, y)\end{aligned}\quad (5)$$

4 EQUILIBRIUM AND STABILITY ANALYSIS

The defuzzified system has three equilibrium categories: the trivial equilibrium $E^0 = (0, 0)$, the axial equilibrium $E^1(x_1, 0) = (k(1 - q_1E^1/\hat{r}), 0)$, and the positive interior equilibrium $E^* = (x^*, y^*)$. The axial equilibrium is biologically meaningful when $\hat{r} > q_1E^1$. The positive equilibrium satisfies Equation (6), and its feasibility follows the conditions reported in the original analysis.

$$\begin{aligned}\hat{r}\left(1 - \frac{x^*}{k}\right) - \frac{p(1-my^*)y^*}{1+ax^*(1-my^*)} - q_1E_1 &= 0 \\ -\hat{d} + \frac{\hat{e}px^*(1-my^*)}{1+ax^*(1-my^*)} - q_2E_2 &= 0\end{aligned}\quad (6)$$

Local asymptotic stability is obtained from the Jacobian matrix. The trivial equilibrium is locally asymptotically stable when $\hat{r} < q_1E^1$ and is a saddle point when $\hat{r} > q_1E^1$. The axial equilibrium is locally asymptotically stable when the harvesting ratio meets Equation (7). For the interior equilibrium, the original conditions require the two inequalities in Equation (8), which jointly constrain predation, refuge, and the effective fuzzy parameters.

$$\left(1 - \frac{\hat{d}+q_2E_2}{\hat{e}pk-ak(\hat{d}+q_2E_2)}\right) < \frac{q_1E_1}{\hat{r}} < 1 \quad (7)$$

$$\begin{aligned}\hat{e}ax^*(1-my^*)^2(p+y^*-px^*) &> py^*(my^*-p-m\hat{e}x^*)-\hat{e}y^* \\ &\quad + \hat{r}x^*(1+ax^*(1-my^*))^2 \\ py^*(1-my^*) &> (1+ax^*(1-my^*))^2\end{aligned}\quad (8)$$

The Jacobian formulation also shows how harvesting, refuge, and the effective biological coefficients enter the local dynamics. Thus, the stability scheme links management variables E_1 and E_2 with the ecological response through the same parameterized system used for the numerical scenarios.

5 SCENARIO-BASED NUMERICAL ASSESSMENT

The numerical assessment uses $k = 5, p = 1.2, a = 0.8, q_1 = 0.4$, and $q_2 = 0.3$. The fuzzy parameters $\tilde{r}, \tilde{d}$, and $\tilde{e}$ are each set to (0.7, 1.0, 1.3). In every scenario, one parameter group is changed while the other baseline parameters remain fixed. Figure 1 and Table 1 report the response to w_1 . Figure 2 and Table 2 report the response to w_2 .

Table 1 Equilibrium Densities and Stability under Different w_1

w_1	x^*	y^*	Stability
0.1	1.856	1.994	LAS
0.5	1.548	1.754	LAS
0.9	1.350	1.541	LAS

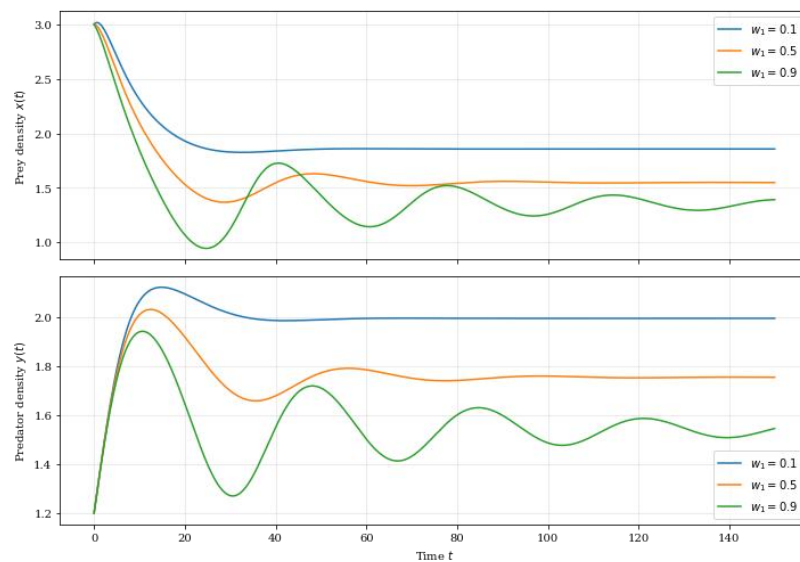


Figure 1 Time-Series Dynamics of the Predator-Prey System under Varying w_1

The source results show that w_1 changes equilibrium densities while preserving local asymptotic stability in the reported cases. The w_2 scenarios modify both effective predator mortality and conversion efficiency; prey equilibrium density increases across the listed cases, while the predator response varies with the altered demographic terms.

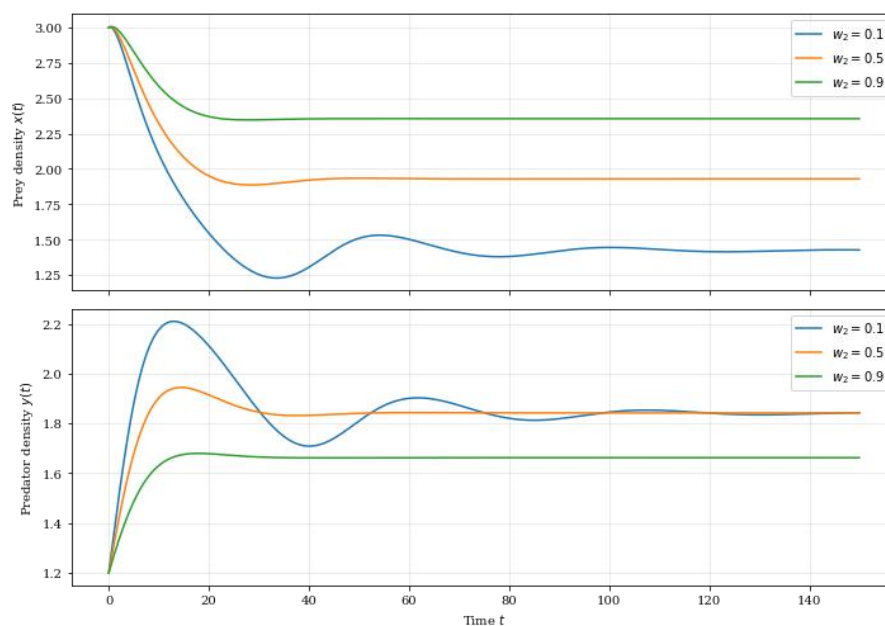


Figure 2 Time-Series Dynamics of the Predator-Prey System under Varying Utility Weight w_2

Table 2 Equilibrium Densities and Stability under Different w_2

w_2	x^*	y^*	Stability
0.1	1.423	1.840	LAS
0.5	1.930	1.842	LAS
0.9	2.356	1.662	LAS

Figure 3 and Table 3 evaluate the uncertainty level ω . Small ω values yield an unstable interior equilibrium, whereas stability is restored in the reported case at $\omega = 1.0$. This comparison directly characterizes the role of the fuzziness level in the defuzzified system.

Table 3 Equilibrium Densities and Stability under Different ω

ω	x^*	y^*	Stability
0.2	0.470	0.915	Unstable
0.6	0.803	1.233	Unstable
1.0	1.802	1.754	LAS

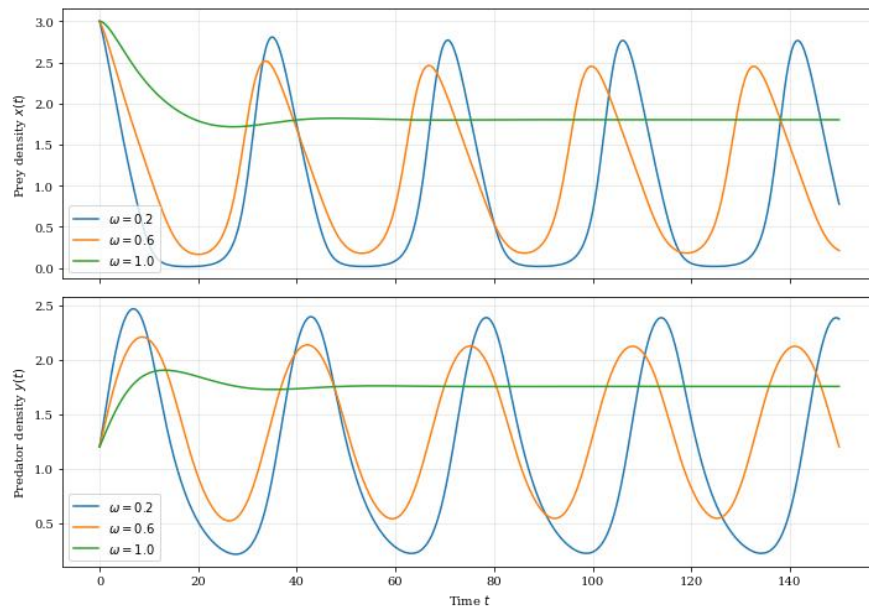


Figure 3 Time-Series Dynamics of the Predator-Prey System under Varying ω

The refuge parameter is assessed in Figure 4 and Table 4. The coexistence equilibrium is unstable without refuge in the source scenario, while positive refuge values produce local asymptotic stability. The reported results therefore identify a stabilizing threshold effect for the refuge mechanism.

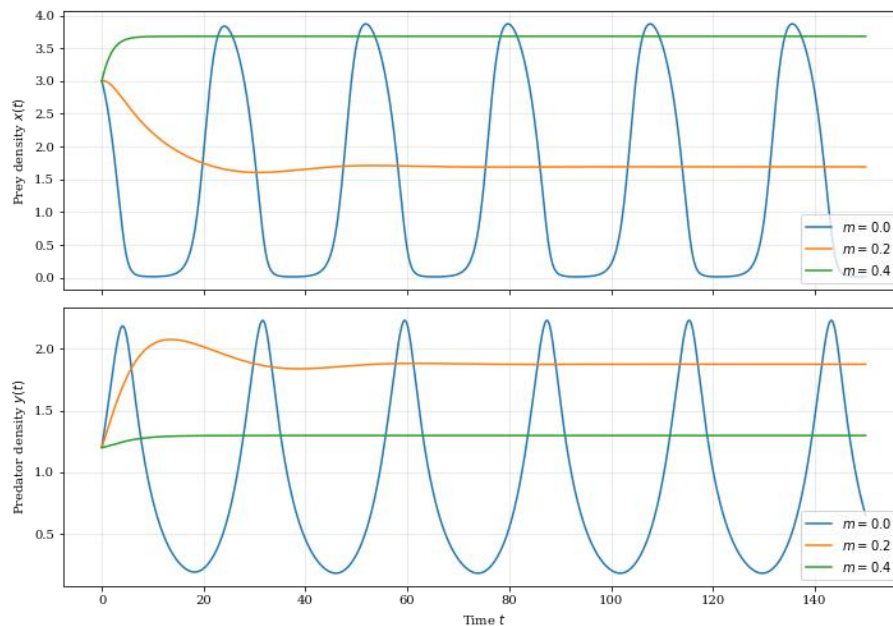


Figure 4 Time-Series Dynamics of the Predator-Prey System under Varying Prey Refuge Parameter m

Table 4 Equilibrium Densities and Stability under Different Refuge Parameter m

m	x^*	y^*	Stability
0.00	0.701	1.041	Unstable
0.20	1.689	1.875	LAS
0.40	3.683	1.297	LAS

Figure 5 and Table 5 present the harvesting scenarios. Under the listed low and moderate harvesting pressures, the populations converge to locally asymptotically stable coexistence equilibria. The source analysis further indicates that higher harvesting intensity can remove the positive coexistence equilibrium and lead to predator extinction.

Table 5 Equilibrium Densities and Stability under Different Harvesting Intensities (E_1, E_2)

(E_1, E_2)	x^*	y^*	Stability
(0.1, 0.05)	2.040	2.299	LAS
(0.3, 0.20)	1.859	1.590	LAS

(E_1, E_2)	x^*	y^*	Stability
(0.6, 0.40)	2.110	0.980	LAS

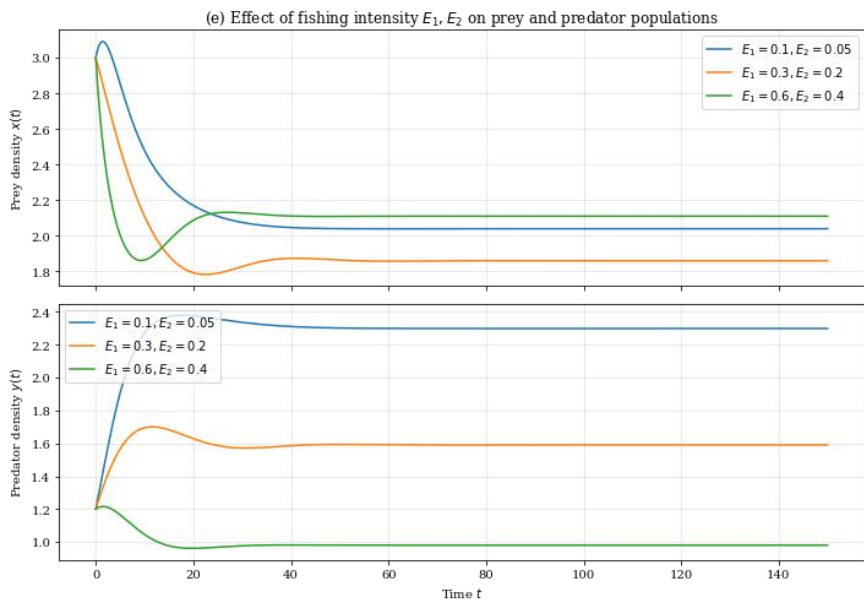


Figure 5 Time-Series Dynamics of the Predator-Prey System under Varying Harvesting Intensities E_1, E_2

6 CONCLUSIONS

This study constructs a harvested predator-prey model with a refuge dependent on both species and with fuzzy biological parameters. The ω -cut and utility-function procedures translate uncertainty in growth, mortality, and conversion efficiency into effective parameters for equilibrium and stability analysis. The source simulations show that the fuzzy weights and ω level alter equilibrium densities and can change coexistence stability. Appropriate prey refuge supports stable coexistence, while excessive harvesting can destabilize the system or eliminate populations. The model therefore provides a research-oriented analytical scheme for assessing sustainable harvesting under ecological parameter uncertainty. Several limitations should be noted. The present analysis is restricted to local stability around equilibria and to a low-dimensional defuzzified system, and it does not explicitly consider global bifurcations, spatial heterogeneity, stochastic forcing, or calibration with field observations. Future work should extend the framework to stochastic and spatial predator-prey dynamics, examine global stability and bifurcation thresholds, and calibrate the uncertain parameters with empirical fishery data so that adaptive harvesting policies can be evaluated under time-varying environmental conditions.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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