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MODELING AND RELIABILITY ANALYSIS OF MULTI-STRESS COUPLED ACCELERATED LIFE MODEL BASED ON FUZZY INFORMATION FUSION

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Abstract: To address the challenges in identifying and accurately characterizing coupling effects in existing multi-stress acceleration models, this paper adopts fuzzy mathematics to characterize uncertainties in both empirical knowledge and data, and proposes a multi-stress accelerated life evaluation method that explicitly accounts for coupling effects. The method first adopts the Distance Correlation Coefficient (DCOR) technique to establish a preliminary screening framework for coupling effects. The objective statistical analysis results are then integrated with subjective empirical judgments via the Fuzzy Number–Entropy Weight–TOPSIS fusion method, yielding an importance ranking of the coupling effects. The selected and optimized acceleration model is subsequently embedded into the log-likelihood function of the Weibull distribution, and the Particle Swarm Optimization (PSO) algorithm is utilized to estimate model parameters and predict service life. Case study results demonstrate that, compared with the traditional Pearson correlation coefficient method, the proposed method exhibits a broader applicability under nonlinear and non-monotonic conditions and significantly improves the prediction accuracy of product service life under constant stress conditions, thereby validating the effectiveness of the model.
Keywords: Accelerated life testing; Multi-stress coupling; Correlation coefficient; Fuzzy number; Particle swarm optimization (PSO) algorithm

1 INTRODUCTION

Modern products are typically characterized by long service lives and high reliability. Accelerated life testing (ALT) simulates harsh operating environments to accelerate the failure process, thereby providing essential data for product reliability analysis [1]. In practice, however, product failures over the lifecycle often result from the combined effects of multiple environmental stresses. To reproduce real-world conditions more accurately, multi-stress accelerated life testing (ALTS) has been developed. Multi-stress acceleration models can transform test data collected under diverse environmental conditions into equivalent data that represent actual service environments [1]. Two widely adopted statistical acceleration models are the generalized log-linear model and the polynomial model [2-3]. To accommodate varying data types and sample sizes, a variety of accelerated modeling approaches have emerged, such as random-effects models, proportional-hazards models, fuzzy prediction models, and support vector machine models [4-6]. Nevertheless, most existing studies do not fully account for the coupling effects among stresses, thereby limiting the applicability of current models in complex stress scenarios.

Accurate characterization of stress coupling effects in multi-stress acceleration models has emerged as a prominent research focus in reliability engineering. In recent years, significant progress has been made in the quantitative characterization of such coupling effects. Luan et al. constructed a stress coupling expression matrix and, using a total differential formulation, demonstrated the inadequacy of simply representing coupling terms as the product of main effects [7]. Building on this, Ye et al. designed a multi-stress acceleration model that employed sets of exponential and power-law candidates as coupling terms, thereby expanding the range of possible expressions for coupling effects [8]. Hou et al. proposed a nonlinear polynomial acceleration model incorporating empirical coupling terms for non-thermal stresses, based on dual-electric stress theory [3]. However, most of the aforementioned studies assume that coupling effects exist between different stresses, an assumption that is clearly unsuitable for high-dimensional stress scenarios. Zhang et al. developed a multi-stress acceleration model with a set of candidate coupling terms derived from stress–failure mode mapping relationships [9]; nevertheless, the selection of coupling effects in that work relied heavily on expert judgment. The MANOVA–Pearson correlation analysis method, commonly adopted in recent years, relies on the assumptions of orthogonal experimental design and normally distributed data [10]. Consequently, it can only capture linear dependencies among variables and is unable to identify nonlinear or non-monotonic coupling effects. In addition, multi-stress accelerated test data may suffer from skewness, heterogeneity, and unknown prior distributions, which can reduce the statistical power of traditional parametric tests [11]. To address these limitations, Zhang et al. integrated the distance correlation coefficient with a two-step analysis approach to propose a model-free method for screening coupling effect variables, and they validated the robustness of this method in the biomedical field [12].

Overall, the identification of significant coupling effects under multi-stress accelerated conditions predominantly relies on subjective experience. This reliance makes it difficult to account for the diversity of failure modes across different products and to accurately characterize the correlative features of coupled failures. Although objective statistical analysis can provide an intuitive explanation of the impact of coupling effects, it typically requires a sufficiently large sample size and the pre-specification of coupling term forms for subsequent verification, thereby introducing additional uncertainty into the results. Fuzzy theory constitutes a key approach for addressing uncertainty. Current research primarily explores the fuzzy treatment of reliability-related indicators from various perspectives—such as accelerated models, acceleration factors, and failure mode and effects analysis—effectively capturing the uncertainty information embedded in these models [13-14]. Therefore, fuzzy theory can be leveraged to handle uncertainties in the modeling of multi-stress acceleration models that involve coupling effects, thus enhancing the model's capacity to incorporate subjective experience and incomplete information.

To address the limitations of most current studies—which neglect uncertainties in the coupling effect screening process and fail to identify nonlinear, non-monotonic coupling terms, this paper employs the Distance Correlation Coefficient (DCOR) method for statistical inference on stress coupling effects in multi-stress acceleration models. By introducing the Fuzzy Number–Entropy Weight–TOPSIS fusion method, a comprehensive ranking of significant coupling effects is derived from the perspective of integrating subjective and objective information. Finally, a stress acceleration model tailored to engineering requirements is constructed to predict product reliability life under normal stress conditions.

2 MULTI-STRESS ACCELERATION MODEL INCORPORATING STRESS COUPLING EFFECTS

2.1 Generalized Linear Acceleration Model Under Multi-Stress Conditions

Currently, the single-stress acceleration models that are most widely used include the Arrhenius model and the inverse power-law model. Among these, the Arrhenius model is

$$\varsigma = \alpha_0 \exp\left(\frac{E_a}{k_B T}\right), \tag{1}$$

Here, ς denotes the lifetime characteristic value of the product, T is the thermal stress, α_0 is a pre-exponential constant, E_a is the activation energy of the chemical reaction, and k_B is the Boltzmann constant. More generally, for a single-stress acceleration model, let $\varphi(S)$ be a function of a single stress variable S, and let β be a model coefficient. Taking the logarithm of Equation (1), the simplified form of the model can be written as

$$\ln\varsigma = \beta\varphi(T) = \beta\frac{1}{T}. \tag{2}$$

Similarly, based on other common expressions for single-stress acceleration models and standard function forms such as exponential, power-law, and logarithmic functions, $\varphi(S)$ primarily takes four classical forms: $S, 1/S, \exp S, \ln S$. Let X be the specific value of $\varphi(S)$ at the single-stress level, $x_{\min}$ represents the stress function value at the normal reference level, $x_{\max}$ represents the stress function value at the ultimate acceleration level, assuming consistent failure mechanisms. The normalization procedure for $\varphi(S)$ is as follows

$$\varphi(S) = (X - x_{\min})/(x_{\max} - x_{\min}). \tag{3}$$

Considering N types of environmental stresses $S = (S_1, \cdots, S_N)$, the generalized Eyring model is commonly used to describe the relationship among temperature, other stresses, and life characteristics. This multi-stress acceleration model can be expressed as

$$\ln\varsigma = \alpha_0 + \beta_1\varphi_1(S_1) + \cdots + \beta_N\varphi_N(S_N). \tag{4}$$

Equation (4) disregards the interactions among stresses. To address this, a generalized linear multi-stress acceleration model incorporating an explicit coupling term is developed in this paper, namely

$$\begin{gathered}\ln\varsigma = \alpha_0 + \sum_{r=1}^{N}\beta_r\varphi_r(S_r) + \sum_{\substack{u=1 \\ 1\le i<j\le N}}^{C_N^2}\delta_{2u}\xi_{2u}^{(2)}(S_i, S_j) + \cdots + \sum_{\substack{u=1 \\ 1\le i<\cdots<v\le N}}^{C_N^k}\delta_{ku}\xi_{ku}^{(k)}(S_i, \cdots, S_v) + \\ \cdots + \delta_N\xi_N^{(N)}(S_1, \cdots, S_N),\end{gathered} \tag{5}$$

here, $\varphi_r(\bullet)$ denotes the main effect function, $\xi_{ku}^{(k)}(\bullet)$ denotes the kth-order coupling effect function, α_0 represents the constant term coefficient, $\beta_1\cdots\beta_N$ represents the main effect coefficient, and $\delta_{k1}\cdots\delta_{ku}$ represents the Kth-order coupling coefficient. The main effect function captures the influence of each stress acting independently on failure time, while the coupling effect function, conceptualized as the interactive superposition of the main effect functions, reflects the impact of stress coupling on product life.

The specific form of the coupling term in a multi-stress generalized linear acceleration model should be determined by actual engineering conditions and the product's inherent failure mechanisms, rather than by the conventional assumption that it directly equals the product of main effect functions. Here, we introduce an extended expression to characterize the coupling effect, whereby the coupling term $\xi_{ku}^{(k)}(\bullet)$ can be expressed as a product of the four classical forms of $\varphi(S)$, namely

$$\xi_{ku}^{(k)}(\bullet) = \prod_{k_1=1}^{K_1} S_{k_1} \times \prod_{k_2=1}^{K_2} \ln S_{k_2} \times \prod_{k_3=1}^{K_3} 1/S_{k_3} \times \prod_{k_4=1}^{K_4} \exp S_{k_4}, k = K_1 + K_2 + K_3 + K_4 \tag{6}$$

2.2 Identification of Multi-Stress Coupling Effects Using the Distance Correlation Coefficient Method

Incorporating all main and coupling effects into an acceleration model would lead to excessive complexity and redundancy. To simplify the model while improving its accuracy, it is necessary to identify the coupling effects that are truly significant. Two types of methods are primarily used for this purpose: subjective empirical methods and objective analytical methods [9, 15]. Traditional objective analytical methods are generally limited to linear scenarios and fail to capture the nonlinear and non-monotonic behaviors observed under real operating conditions. Even when the main stress effects in the acceleration model follow linear laws, coupling terms that deviate from simple product forms may still exhibit nonlinear relationships.

The distance correlation coefficient (DCOR) is a nonparametric measure of association that does not rely on model assumptions. It computes correlations based on the distance matrix among data points, without requiring the data to follow a specific distribution, and can effectively capture overall linear, nonlinear, or non-monotonic time-varying cumulative relationships among variables. Accordingly, this paper introduces the DCOR method to assess the influence of main effects and coupling effects on failure time, and identifies significant coupling effects from the main effects based on genetic effects [12]. Let x and y denote two random variables. The distance correlation coefficient between x and y is defined as follows:

$$dcorr^2(x,y) = \frac{d\,cov^2(x,y)}{\sqrt{d\,cov^2(x,x)d\,cov^2(y,y)}},$$
$$d\,cov^2(x,y) = E(\|x-\bar{x}\|\cdot\|y-\bar{y}\|) + E(\|x-\bar{x}\|)\cdot E(\|y-\bar{y}\|) - 2E\big(E(\|x-\bar{x}\||x)\cdot E(\|y-\bar{y}\||y)\big), \tag{7}$$

here, $\|\bullet\|$ represents the Euclidean norm, and $\bar{x}$、$\bar{y}$ is an independent and identically distributed (i.i.d.) copy of x、y. The distance correlation coefficient, computed through $\varphi(S)$, ξ, and $\ln\varsigma$, can be expressed in terms of $dcorr^2(\varphi(S),\ln\varsigma)$ and $dcorr^2(\xi,\ln\varsigma)$. This coefficient reflects the degree of correlation between the coupling effect and failure time. If the coupling effect is not significant, $dcorr^2$ should be close to 0; conversely, a larger value of $dcorr^2$ indicates a stronger influence on failure time. In general, if the condition $dcorr^2 > 0.4$ holds, the coupling effect is considered significant.

In addition, to determine the optimal form of the coupling term, based on Equations (6) and (7), the distance correlation coefficient between ξ and $\ln\varsigma$ can be computed for different candidate expressions through iterative traversal. This enables objective identification of the coupling effect and determination of the optimal coupling term form.

Numerical simulations are presented below to investigate the relationship between multi-stress coupling effects and failure time, thereby verifying the applicability and validity of the proposed method.

2.2.1 Advantages of the DCOR method for identifying nonlinear and non-monotonic relationships

Environmental stress and product lifetime often exhibit a complex, nonlinear, and non-monotonic relationship. For instance, certain electronic components have an optimal operating range, and their lifetime decreases when they deviate from this range. Assuming that a given electronic component undergoes synergistic failure under coupled temperature–humidity conditions, a nonlinear acceleration model was constructed based on the operating conditions:

$$\ln t = 9.21 - 0.5\left(\frac{T-298.15}{25}\right)^2 - 4Z, Z = \left(\frac{T-298.15}{25}\cdot\frac{RH-50}{20}\right). \tag{8}$$

Here, $\ln t$ represents the logarithmic lifetime, while T、RH、Z represent temperature stress, humidity stress, and the temperature–humidity coupling term, respectively. The temperature range was set to 273.15K–333.15 K and the humidity range to 20%–80%, with the optimal operating environment being 298.15 K and 50%. Figure 1 illustrates the variation of the response surface as a function of temperature and humidity. The surface exhibits a distinct inverted U-shape, and the contour lines follow a hyperbolic distribution, indicating that the rate of lifetime decay under coupled stresses is faster than that under the influence of a single stress. This suggests a multiplicative coupling effect between temperature and humidity and validates the symmetric nonlinear relationship between the coupling effect Z and failure time. The correlation coefficients obtained using the DCOR method are significantly higher. This result indicates that when variables exhibit a nonlinear relationship, the Pearson correlation coefficient may underestimate the strength of the association, whereas the distance correlation coefficient can effectively capture such dependencies.

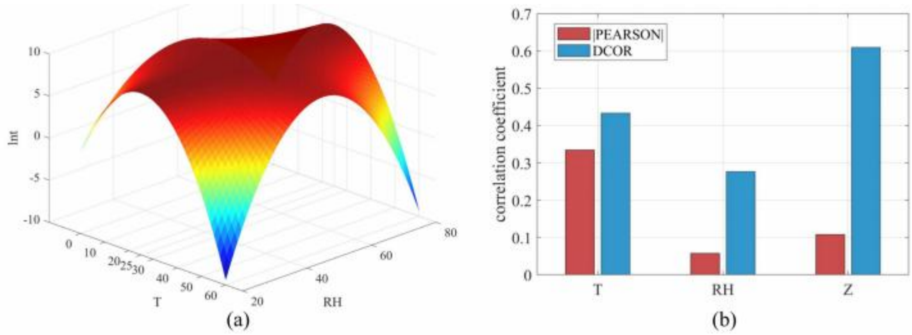

Figure 1 (a) 3D Response Surface Plots for Each Variable (b) Comparison of Results Obtained Using the Two Correlation Coefficient Methods

2.2.2 Effectiveness of the DCOR method in identifying nonlinear monotonic relationships

For validation, the polynomial multi-stress acceleration model proposed in [8] is adopted. This model is expressed as

$$\varsigma = \alpha_1 x_1^{\beta_1} + \alpha_2 x_2^{\beta_2} + \alpha_3 \ln(x_1)e^{x_2}. \tag{9}$$

Here, ς、x_1、x_2 denote the battery's cycle life, SOC stress, and C2 stress, respectively, and α_1、α_1、α_1、β_1、β_2 represent the model parameters. The model was reconstructed using the parameter estimates from the original paper, with noise of magnitude 0.1 added to generate uniformly random data. The results indicate that x_2、$\ln(x_1)e^{x_2}$ and ς

are highly correlated, which is consistent with the sensitivity analysis. Taking the second subfigure in the second row of Figure 2 as an example, the scatter plot exhibits a curvilinear increasing trend, and the two methods yield similar correlation results, indirectly demonstrating the capability of the DCOR method to handle weakly nonlinear monotonic relationships.

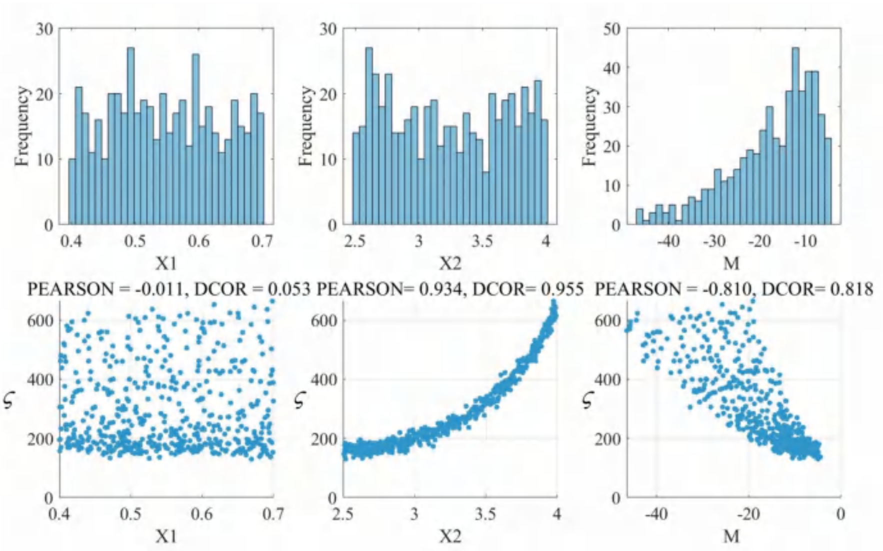

Figure 2 Distribution Plots of Various Effects and Scatter Plots of Lifetime Characteristics

3 SCREENING OF SIGNIFICANT COUPLING EFFECTS BASED ON THE FUZZY NUMBER-ENTROPY WEIGHT-TOPSIS FUSION METHOD

After effectively identifying the coupling effects, it is necessary to screen and determine which significant coupling effects should be incorporated into the acceleration model, thereby reducing the computational burden of subsequent parameter estimation. Based on Equation (5), a selection coefficient $\lambda \in \{0, 1\}$ is introduced to decide whether a coupling term should be included in the acceleration model: $\lambda_u = 1$ indicates inclusion, and otherwise, it is excluded. The model can be intuitively expressed as

$$\ln\varsigma = \alpha_0 + \sum_{r=1}^{N} \beta_r \varphi_r(S_r) + \sum_{\substack{1 \le i < j \le N \\ u=1}}^{C_N^2} \lambda_{2u} \delta_{2u} \xi_{2u}^{(2)}(S_i, S_j) + \cdots + \sum_{\substack{1 \le i < \cdots < v \le N \\ u=1}}^{C_N^k} \lambda_{ku} \delta_{ku} \xi_{ku}^{(k)}(S_i, \cdots, S_v) + \cdots + \lambda_N \delta_N \xi_N^{(N)}(S_1, \cdots, S_N), \tag{10}$$

Therefore, it is necessary to determine the values of λ for each term in advance, that is, to first identify the significant coupling terms to be retained in the model before estimating the model coefficients. Given the limitations of objective methods in identifying coupling effects, a fuzzy information fusion method is introduced to integrate subjective expert judgments, thereby enabling a comprehensive screening of significant coupling effects based on both objective and subjective criteria. This method draws upon the comprehensive evaluation framework that combines fuzzy numbers, entropy weighting, and the TOPSIS technique. It uniformly quantifies the fuzziness of subjective experience and the randomness of data results, employs membership functions to characterize the fuzzy attributes of each indicator, performs weight calibration through entropy-based fusion, and ultimately ranks and determines the coupling stress groups to be included in the acceleration model via the TOPSIS method.

3.1 Construction of Integrated Information and Fuzzy Evaluation Matrix

Let $A = (A_1, A_2, \cdots A_n)$ represent the evaluation objects, namely, the coupling effects to be screened through both subjective and objective methods; let $B = (B_1, B_2, \cdots B_m)$ represent the evaluation criteria, which consist of two types of coupling effect identification methods (subjective and objective). A fuzzy evaluation matrix $\widetilde{Y} = \left[\tilde{y}_{ij}\right]_{n \times m}$ is constructed, where $\tilde{y}_{ij}$ denotes the fuzzy value of the i-th evaluation object under the j-th criterion.

3.2 Establishment of Fuzzy Evaluation Levels

To quantify the impact of uncertainty, a fuzzy number $\tilde{y}_{ij}$ is adopted to represent the degree of influence of the stress coupling effect. Based on the application scenario considered in this paper, two representative evaluation criteria are employed to establish fuzzy evaluation levels and construct the fuzzy evaluation matrix: expert linguistic evaluations serve as the subjective criterion, and the DCOR method serves as the objective criterion. The judgment results are converted into triangular fuzzy numbers and rectangular fuzzy numbers, respectively, yielding the fuzzy evaluation matrix $\widetilde{Y}_{n2}$. The specific procedure is as follows:

Expert linguistic evaluation results are converted into triangular fuzzy numbers by constructing a five-level subjective linguistic evaluation set: L = {"No Impact", "Weak Impact", "General Impact", "Significant impact", "Strong impact"}. Each linguistic level in this set corresponds to a distinct triangular fuzzy number $\tilde{L} = (l^L, l^M, l^U)$, and Figure 3 presents the membership function distributions for each level.

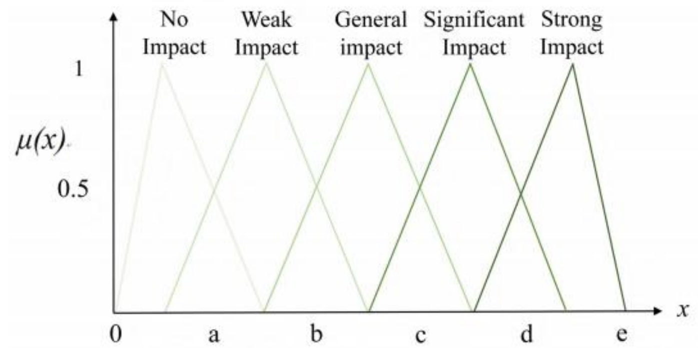

Figure 3 Membership Distributions of Triangular Fuzzy Numbers for Each Subjective Level

Conversion of DCOR method results to trapezoidal fuzzy numbers is performed as follows: by introducing the trapezoidal fuzzy number $\tilde{r} = (r^L, r^{ML}, r^{MU}, r^U)$, the single-point value $dcorr^2$ is expanded into an interval-type fuzzy value to characterize the uncertainty fluctuations in the numerical results. The absolute value $|r|$ is taken as the midpoint of the peak interval of the trapezoidal fuzzy number, and a fuzzy radius is introduced to quantify the parameter uncertainty. The expression is

$$\tilde{r} = (r^L, r^{ML}, r^{MU}, r^U) = (|r| - \upsilon - \varepsilon, |r| - \varepsilon, |r| + \varepsilon, |r| + \upsilon + \varepsilon), \tag{11}$$

here, ε and $\upsilon + \varepsilon$ denote the half-width of the half-peak interval and the half-width of the support, respectively. To ensure reasonable values, the boundaries of the fuzzy numbers are truncated and corrected as follows:

$$r^L = \max(0, r^L), r^{ML} = \max(0, r^{ML}), r^{MU} = \min(1, r^{MU}), r^U = \min(1, r^U). \tag{12}$$

Based on the fuzzy number construction strategies and transformation methods described above, the fuzzy evaluation matrix $\tilde{Y}_{n2}$ can be further established as follows:

$$\tilde{Y}_{n2} = \begin{bmatrix} \tilde{y}_{11} & \cdots & \tilde{y}_{1n} \\ \tilde{y}_{21} & \cdots & \tilde{y}_{2n} \end{bmatrix}^{\mathrm{T}}, \tilde{y}_{ij} = \begin{cases} (l^L, l^M, l^U), B_j \in \text{subjective method} \\ (r^L, r^{ML}, r^{MU}, r^U), B_j \in \text{objective method} \end{cases}. \tag{13}$$

3.3 Construction of the Weighted Fuzzy Comprehensive Evaluation Matrix

To reduce the bias introduced by subjective weighting, the entropy-weighted method is adopted below to derive a weighted fuzzy comprehensive evaluation matrix. First, the matrix $\tilde{Y}_{n2}$ is subjected to dimensionless normalization, yielding the normalized fuzzy matrix $\tilde{Y}_{n2}^{'} = \left[\tilde{y}_{ij}^{'}\right]_{n\times 2}$, where M_j denotes the normalization reference value. The normalization formula is as follows:

$$\tilde{y}_{ij}^{'} = \frac{\tilde{y}_{ij}}{M_j} = \begin{cases} \left(\frac{l^L}{M_j}, \frac{l^M}{M_j}, \frac{l^U}{M_j}\right), B_j \in \text{subjective method} \\ \left(\frac{r^L}{M_j}, \frac{r^{ML}}{M_j}, \frac{r^{MU}}{M_j}, \frac{r^U}{M_j}\right), B_j \in \text{objective method} \end{cases}, M_j = \max\{\tilde{y}_{ij} | i = 1、2、\cdots n\}. \tag{14}$$

Second, the center-of-mass method is applied to defuzzify the fuzzy number $\tilde{y}_{ij}^{'}$ into a crisp value $y_{ij}^{'}$ suitable for entropy weight calculation, where $C(\bullet)$ denotes the centroid operator and $\mu(x)$ denotes the membership function of the corresponding fuzzy number. The defuzzification formula is as follows:

$$y_{ij}^{'} = C\left(\tilde{y}_{ij}^{'}\right), C(\bullet) = \int_{-\infty}^{+\infty} x\mu(x)dx / \int_{-\infty}^{+\infty} \mu(x)dx. \tag{15}$$

Next, the information entropy H_j corresponding to each B_j is computed in turn, and the weight vector $W_H = \left(w_H^1, w_H^2\right)$ is determined. Subsequently, the weighted fuzzy comprehensive evaluation matrix $\tilde{Z}_{n2} = \left[\tilde{z}_{ij}\right]_{n\times 2}$ is constructed, where the elements are given by $\tilde{z}_{ij} = \tilde{y}_{ij}^{'} \times w_H^j$. The calculation formulas are as follows:

$$H_j = -k\sum_{i=1}^{n} p_{ij}\ln p_{ij}, p_{ij} = y_{ij}^{'} / \sum_{i=1}^{n} y_{ij}^{'}, k = 1/\ln n, w_H^j = \left(1 - H_j\right) / \sum_{j=1}^{2}\left(1 - H_j\right). \tag{16}$$

3.4 Quantifying and Ranking the Importance of Coupling Effects

Finally, the TOPSIS method is applied to rank the coupling effects by their degree of influence, thereby identifying the key coupling effects to be retained in the model. The ranking criterion requires that the evaluation object be closest to the positive ideal solution E^+ and farthest from the negative ideal solution E^-. The positive and negative ideal solutions are respectively expressed as follows:

$$E^+ = (z_1^+, z_2^+), z_j^+ = \max_i z_{ij};\ E^- = (z_1^-, z_2^-), z_j^- = \min_i z_{ij}. \tag{17}$$

Here, the Euclidean distance is employed to compute the distances D_i^+ and D_i^- of each evaluation object to the positive and negative ideal solutions, respectively, as well as the relative proximity C_i of each evaluation object. The calculations are as follows:

$$D_i^+ = \sqrt{\sum_{j=1}^{m}\left(z_{ij}-z_j^+\right)^2}, D_i^- = \sqrt{\sum_{j=1}^{m}\left(z_{ij}-z_j^-\right)^2}, C_i = \frac{D_i^-}{D_i^- + D_i^+}, i = 1,2,\cdots,n, \tag{18}$$

the larger the value of CCC_i, the higher the priority that should be assigned to the corresponding coupling effect in the acceleration model. Accordingly, the discrimination coefficient λ_u in Equation (10) can be specifically expressed as

$$\lambda_u = \begin{cases} 1, C_u > 0.5 \\ 0, C_u \leq 0.5 \end{cases}. \tag{19}$$

4 PARAMETER ESTIMATION METHOD FOR THE MULTI-STRESS ACCELERATION MODEL

After deriving a multi-stress acceleration model that incorporates significant coupling effects, the stress level settings and ALT data were used as references to obtain maximum likelihood estimates of the unknown model parameters via the nonlinear particle swarm optimization (PSO) method. In fields such as reliability engineering, survival analysis, and risk assessment, the lifetime of components, equipment, or systems that become inoperable due to local failures can be assumed to follow a Weibull distribution [16]. The corresponding distribution function and reliability function are, respectively,

$$F(t;m,\eta) = 1 - \exp\left(-\left(\frac{t}{\eta}\right)^m\right), R(t;m,\eta) = \exp\left(-\left(\frac{t}{\eta}\right)^m\right), t > 0, \tag{20}$$

in this equation, m denotes the shape parameter, and η denotes the scale parameter (or characteristic lifetime). A complete failure test is defined as one that continues until all samples have failed. Next, a statistical analysis is conducted using Weibull-type failure time data from constant-load testing as an example, based on the following assumptions:

A1. Product lifetimes under both constant stress and accelerated stress follow Weibull distribution;

A2. The failure mechanism remains unchanged under normal and accelerated stress conditions, i.e.,

$$m_0 = m_1 = \cdots = m_k; \tag{21}$$

A3. The relationship between the applied stress S and the characteristic lifetime η satisfies the acceleration model.

Table 1 Particle Swarm Optimization Algorithmic Steps

Step1: Initialization
The initial parameter estimates obtained from least-squares regression are used as prior centers to generate a set of quantitative initial particles. If prior parameter values are unavailable, Latin Hypercube Sampling (LHS) is adopted instead. The maximum particle velocity is set to 20% of the interval width.
Step2: Fitness Evaluation
The log-likelihood function serves as the fitness function, with the objective of maximizing the target value. The fitness value of each particle is computed accordingly.
Step3: Individual and Global Best Updates
For each particle, the all-time best individual position p_{best} is recorded; for the entire population, the global best position g_{best} is updated.
Step4: Velocity and Position Updates
The velocity and position of each particle are updated iteratively according to the following formula: $$v_i^{t+1} = wv_i^t + c_1 r_1 (p_{best,i} - x_i^t) + c_2 r_2 (g_{best,i} - x_i^t), x_i^{t+1} = x_i^t + v_i^{t+1}, \quad w = w_{\max} - (w_{\max} - w_{\min})(1 - (\frac{t}{t_{\max}} - 1)^2), \tag{25}$$ where w denotes the inertia weight, c_1、c_2 denote the acceleration coefficient, r_1、r_2 are random number, t and $t_{\max}$ represent the current iteration number and the maximum number of iterations, respectively.
Step5: Boundary Constraint Handling
For any particle whose position exceeds the search boundary, a mirror reflection is first applied, and the resulting value is then truncated to the feasible range.
Step6: Termination Check
Steps 1 through 5 are repeated until the maximum number of iterations is reached or the change in fitness falls below a preset threshold.

Suppose a multi-stress accelerated life test consists of k independent stress-level combinations, each involving N types of environmental stresses and n test products. For the h-th stress-level combination, let the failure times of the n products be sorted in ascending order and denoted as $t_{h1},\cdots,t_{hn_h}$. The likelihood function corresponding to this group is

$$L(t_h;m_h,\eta_h) = \prod_{i=1}^{n_h} \frac{m_h}{(\eta_h)^m}\frac{{t_{hi}}^{m_h-1}}{\eta_h}\exp\left(-\left(\frac{t_{hi}}{\eta_h}\right)^{m_h}\right), t > 0. \tag{22}$$

Taking the natural logarithm of Equation (22) yields the log-likelihood function for the h-th group as

$$\begin{aligned}\ln L_h(t_h;m_h,\eta_h) &= \ln\left(\prod_{i=1}^{n_h}\frac{m_h}{(\eta_h)^{m_h}}\left(\frac{t_{hi}}{\eta_h}\right)^{m_h-1}\exp\left(-\left(\frac{t_{hi}}{\eta_h}\right)^{m_h}\right)\right.\\ &= n_h\ln m_h - n_h m_h \ln\eta_h + (m_h-1)\sum_{i=1}^{n_h}\ln t_{hi} - \frac{1}{(\eta_h)^{m_h}}\sum_{i=1}^{n_h} t_{hi}^{m_h}, t>0.\end{aligned} \tag{23}$$

Since the test samples across groups are independent, the log-likelihood functions for all test groups are summed according to Equation (23), and the expression for η from Equation (7) is substituted to obtain the overall log-likelihood function:

$$L(m,\alpha_0,\beta,\delta) = \sum_{h=1}^{k}\ln L_h(t_h;m_h,\eta_h), t>0, \tag{24}$$

here, $m=(m_1,\cdots,m_k),\alpha_0,\beta=(\beta_1,\cdots,\beta_N),\delta=(\delta_1,\cdots,\delta_u)$ denote the shape parameter, the constant term, and the parameters associated with the main and coupling effects, respectively, in the acceleration model for each set of experiments. By maximizing Equation (24), the maximum likelihood estimates of all unknown parameters can be obtained. This paper adopts the PSO algorithm to address such high-dimensional constrained problems and to search for the model parameter values. Following Reference [16], the model parameters and boundary constraints are initialized using prior information, and an adaptive nonlinear coefficient is introduced to balance the global search and local convergence capabilities of the algorithm. Table 1 presents the algorithmic steps.

5 CASE ANALYSIS

Taking as an example the pseudo-lifetime data obtained from accelerated degradation tests focusing on the performance-sensitive parameters of a specific type of smart meter, this paper validates the proposed method for identifying and screening coupling effects. The experiment comprised five independent groups of multi-stress accelerated life tests. In each group, three environmental stresses—temperature, humidity, and current—were simultaneously applied to 54 products from the same batch [17]. The pseudo-lifetime of this type of smart meter is known to follow a Weibull distribution. Table 2 lists the stress-level settings used in the accelerated life tests.

Table 2 Setting of Stress Levels for Accelerated Life Testing of Smart Meters

Group	Temperature (T/°C)	Humidity (RH/%)	Current (I/A)	Number of Samples
1	80	80	60	54
2	55	80	40	54
3	55	95	20	54
4	70	95	60	54
5	70	95	40	54

In the preliminary stage of this study, an analysis of stress–failure mechanisms identified measurement error as the core failure mode for this type of smart meter [18]. Empirical analysis indicates that temperature and current exert bidirectional and synergistic effects on failure: temperature amplifies the destructive effect of humidity on chip packaging and internal metal components; humidity indirectly influences current distribution by altering the resistive properties of materials; and accelerated testing results show that temperature and current affect the metering errors of smart meters, while humidity stress has a relatively weak impact on this failure mode. In summary, subjective empirical methods suggest the presence of two coupling effects, namely $\xi(T,I)$、$\xi(T,RH)$.

Based on Equation (7), the DCOR method was applied to explore potential coupling effects. The selection or rejection of coupling terms was guided by the magnitude of the correlation coefficients between each effect and lifetime characteristics, in conjunction with the underlying physical mechanisms. According to [17], all main effect functions assume a reciprocal form. Figure 4 presents scatter plots of the coupling terms against lifetime characteristics. In the small-sample scenario, the data points in each sub-plot display curved or inverted V-shaped distributions rather than linear alignment; correspondingly, the distance correlation coefficients for each coupling term are consistently higher than the Pearson correlation coefficients, further quantifying the strength of the nonlinear dependence. It can be preliminarily concluded that $\xi(T,RH)$ may have a significant impact on the lifespan of smart meters.

Combining the subjective expert judgments and objective statistical analysis results presented in Table 3, fuzzy information fusion is performed as follows: Given $\varepsilon=0.025$, $\upsilon=0.1$, a fuzzy evaluation matrix is constructed based on Equations (11) through (16), with the four coupling effects serving as evaluation objects and the two benefit-type indicators (subjective and objective) as evaluation criteria. This yields the objective weight $W_H=(0.808,0.193)$ and the weighted fuzzy comprehensive evaluation matrix $\widetilde{Z}$, namely

$$\widetilde{Z} = \begin{bmatrix}(0.269,0.449,0.628) & (0.142,0.162,0.172,0.193)\\ (0.449,0.628,0.808) & (0.084,0.104,0.114,0.134)\\ (0.090,0.269,0.449) & (0.090,0.111,0.121,0.141)\\ (0.090,0.269,0.449) & (0.094,0.114,0.124,0.144)\end{bmatrix}. \tag{26}$$

Based on Equation (18), the distances to the positive and negative ideal solutions, as well as the closeness coefficients for each evaluation object, were calculated, and the coupling effects were ultimately ranked in descending order of their influence as follows:

$$\xi(T,I)(0.859) > \xi(T,RH)(0.513) > \xi(RH,I)(0.028) > \xi(T,RH,I)(0.019). \tag{27}$$

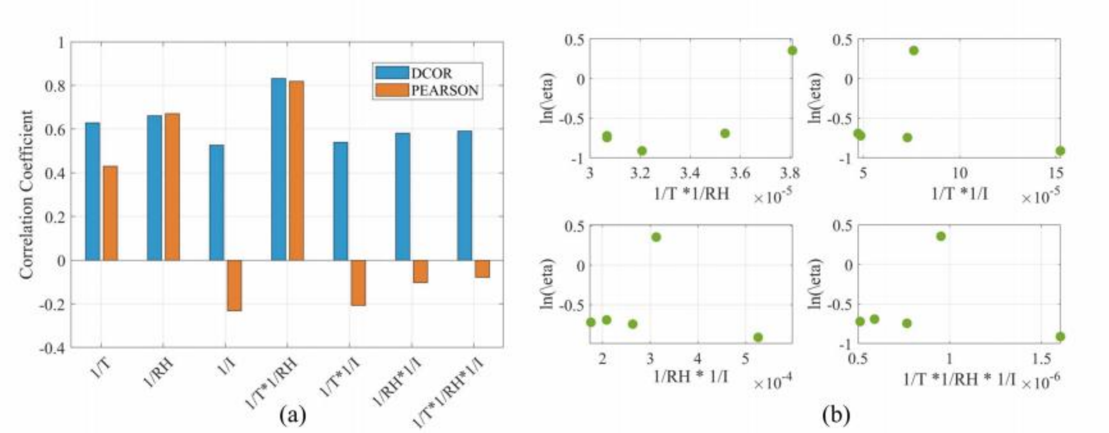

Figure 4 (a) Comparison of Results From the Two Types of Correlation Coefficient Methods (b) Scatter Plots Showing the Relationship Between Various Effects and Lifespan Characteristics

Table 3 Results of the Assessment of Various Coupling Effects Using Subjective and Objective Methods

Coupling Terms	$\xi(T, RH)$	$\xi(T, I)$	$\xi(RH, I)$	$\xi(T, RH, I)$
Subjective Method	General	Strong	Weak	Weak
Objective Method	0.832	0.540	0.574	0.591

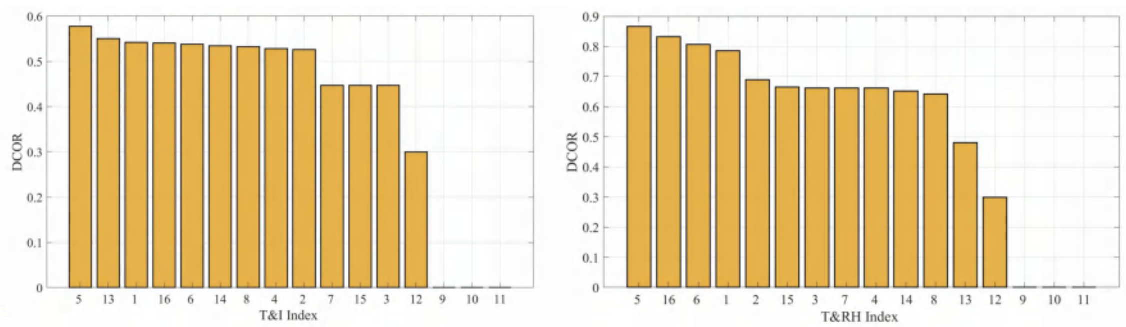

Figure 5 Ranking of Optimal Levels for Various Representations of T&I and T&RH Coupling Terms Under the DCOR Method

Figure 5 indicates that the optimal coupling forms for $\xi(T, RH)$、$\xi(T, I)$ are $T \times \ln RH$、$T \times \ln I$, respectively, which differ from the conventional preset forms. According to Equation (10), the optimal multi-stress generalized linear acceleration model can be modified as

$$\ln\eta = \alpha_0 + \beta_1\left(\frac{1}{T}\right) + \beta_2\left(\frac{1}{RH}\right) + \beta_3\left(\frac{1}{I}\right) + \delta_1(T) \times (\ln RH) + \delta_2(T) \times \ln(I), \tag{28}$$

here, $a_0, \beta_1, \beta_2, \beta_3, \delta_1, \delta_2$ are the parameters to be estimated. Based on the differences in stress coupling relationships, seven comparison models were constructed and optimized using PSO. To eliminate the impact of algorithmic randomness on the results, each model was configured with six parallel optimization paths, all adopting the same core PSO parameter settings: $w_{\max} = 0.8$、$w_{\min} = 0.2$、$c_1 = c_2 = 2.0$, 150 particles, a maximum of 800 iterations for uncoupled terms, and a maximum of 10,000 iterations for coupled terms. The main effects are identical across all models, and the settings of the coupling terms are presented in Table 4.

Table 4 Configurations of Coupling Term Representations for the Seven Comparison Models

model	M1	M2	M3	M4	M5	M6	M7
$\varphi(T, RH)$	/	/	$1/T \times 1/RH$	/	$T \times \ln RH$	$T \times \ln RH$	$1/T \times 1/RH$
$\varphi(T, \ln I)$	/	$1/T \times 1/I$	$1/T \times 1/I$	$T \times \ln I$	$T \times \ln I$	$1/T \times 1/I$	$T \times \ln I$

The results presented in Figure 6 and Table 5 indicate that, regardless of whether coupling effects are considered, the shape parameter estimates of the multi-stress accelerated life model all exceed 2.5. This is consistent with the wear-out failure behavior typical of electronic devices and satisfies the principle of failure mechanism consistency. Moreover, the parameter estimates obtained by the proposed method are relatively stable and, unlike those in Reference [17], do not require subsequent correction. Compared with the biased conclusions drawn from the models in [17-19], the parameter estimates for M3, M5, M6, and M7 obtained in this study align with the actual survey results under normal operating conditions. The AIC results indicate that the accelerated model incorporating coupling effects not only fits better but also yields more conservative predicted lifetimes. In addition, the percentage errors (e%) between the average predicted lifetimes and the actual values were calculated for models M1–M7 under the five sets of test stresses. Figure 6 shows that the errors for M4, M5, and M6 are approximately 2.000%; those for M3 and M7 are approximately 2.300%; and those for M1 and M2 are 6.120% and 2.500%, respectively. All of these are substantially lower than the errors of 22.240% and 9.430% reported for the two model types in Reference [4], demonstrating that the optimized PSO method adopted in this study achieves higher accuracy than traditional approaches and that neither the temperature–humidity nor the temperature–current coupling effect can be neglected. According to the real-world failure data for the batch of 586 smart meters installed in the Daxing District of Beijing, as reported in Reference [17], the actual failure rate after five years of operation is 5.461%. Given the regional annual average temperature of 15°C, average humidity of 40%, and average user current of 10 A, the reliable lifetimes ($t_{0.945}$) predicted by the seven models at a reliability level of R = 94.500% were compared. The result of 5.030 years obtained by model M5 is closest to the actual value and is consistent with the service conditions of the sampled meters in the real environment. Overall, the predictions from the optimal coupling model M5 adopted in this paper better reflect the actual failure behavior of the product. To extend the actual service life to 10 years as required by domestic standards, regular inspections and long-term maintenance are necessary.

Table 5 Parameter Estimates and Significance for Seven Alignment Acceleration Models

MODEL	$\widehat{m_0}$		$\widehat{a_0}$		$\widehat{\beta_1}$		$\widehat{\beta_2}$		$\widehat{\beta_3}$		$\widehat{\delta_1}$		$\widehat{\delta_2}$
M1	3.039	**	2.919	**	-3.840	**	-2.907	**	2.743	**	/		/
M2	3.158	**	-2.174	**	4.884	**	-3.560	**	8.472	**	/		-8.812
M3	3.164	**	2.027	**	0.152		-7.242	**	7.356	**	4.282	**	-7.686
M4	3.177	**	1.095	**	-1.030	**	-3.511	**	4.516	**	/		-2.260
M5	3.177	**	2.520	**	-2.121	**	-4.762	**	4.288	**	1.067	**	-2.142
M6	3.177	**	2.458	**	-2.474	**	-4.825	**	4.286	**	1.535	**	-2.140
M7	3.169	**	2.586	**	0.954		-7.560	**	7.448	**	3.393	**	-7.889

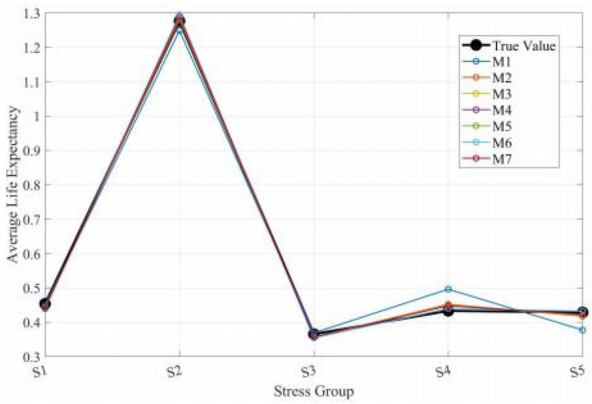

Figure 6 Comparison of Average Lifespan Predictions vs. Actual Values for Different Models Across Each Test Group

6 CONCLUSION

This paper addresses practical engineering problems in which multiple stress couplings affect product failure. Based on uncertainty analysis, a multi-stress generalized linear acceleration model incorporating significant coupling effects is developed to predict service life under steady-state stress conditions. The DCOR method is employed to identify coupling effects and update their characterization forms; compared with the conventional Pearson method, the DCOR method demonstrates notable advantages under non-monotonic and nonlinear conditions. Subjective empirical methods are integrated with objective analytical methods to screen for significant coupling effects. Uncertainty is characterized using fuzzy numbers, and the importance of each coupling effect is quantified by combining the entropy-weighted method with the TOPSIS method, yielding a more comprehensive assessment than that obtained from a single perspective. The PSO algorithm is introduced into the parameter estimation of the Weibull likelihood function to achieve iterative updating of model parameters. Using pseudo-lifetime data from a specific type of smart meter as an example, the analysis shows that after accounting for stress coupling effects, the prediction accuracy of the model improves significantly—the relative error decreases from 22.240% to 2.020%, and the extrapolated lifetime aligns more closely with actual usage conditions—thereby validating the effectiveness of the proposed method. The method proposed in this paper still has certain limitations: the acceleration model is assumed to be linear and incorporates only four coupling factors, which may be insufficient to fully capture the complex mapping relationship between environmental stresses and life characteristics, future work could therefore explore the introduction of additional alternative forms; As for the fuzzy information fusion component, further improvements may be achieved by integrating multiple information sources and introducing preference coefficients to refine the fusion results, thereby enhancing the overall adaptability of the method.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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